

Event-based state estimation: an emulation-based approach

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Abstract: An event-based state estimation approach for reducing communication in a networked control system is proposed. Multiple distributed sensor agents observe a dynamic process and sporadically transmit their measurements to estimator agents over a shared bus network. Local event-triggering protocols ensure that data is transmitted only when necessary to meet a desired estimation accuracy. The event-based design is shown to emulate the performance of a centralised state observer design up to guaranteed bounds, but with reduced communication. The stability results for state estimation are extended to the distributed control system that results when the local estimates are used for feedback control. Results from numerical simulations and hardware experiments illustrate the effectiveness of the proposed approach in reducing network communication.

1 Introduction

In almost all control systems today, data is processed and transferred between the system's components periodically. While periodic system design is often convenient and well understood, it involves an inherent limitation: data is processed and transmitted at predetermined time instants, irrespective of the current state of the system or the information content of the data. That is, system resources are used regardless of whether there is any need for processing and communication or not. This becomes prohibitive when resources are scarce, such as in networked or cyber-physical systems, where multiple agents share a communication medium.

Owing to the limitations of traditional design methodologies for resource-constrained problems, aperiodic or event-based strategies have recently received a lot of attention [1, 2]. With event-based methods, data is transmitted or processed only when certain *events* indicate that an update is required, e.g. to meet some control or estimation specification. Thus, resources are used *only when required* and saved otherwise.

In this paper, a novel event-based scheme for distributed state estimation is proposed. We consider the system shown in Fig. 1, where multiple sensors observe a dynamic system and transmit data to estimator agents over a common bus. Each estimator agent shall estimate the full state of the dynamic system, e.g. for the purpose of monitoring or control. To limit network traffic, local event triggers on each sensor ensure that updates are sent only when needed. The common bus ensures that transmitted data reaches all agents in the network, which will allow for efficient

triggering decisions and the availability of full state information on all agents.

The proposed approach for distributed event-based estimation emulates a classic, discrete-time state observer design up to guaranteed bounds, but with limited communication. *Emulation-based design* is one common approach in event-based control literature (see [2]), where an event-based control system is designed so as to emulate the behaviour of a given continuous or periodic control system. However, to the best of the author's knowledge, emulation-based design has not been considered for state estimation. While the focus of this paper is on state estimation, we also show stability of the event-based control system resulting when local estimates are used for feedback control.

In particular, this paper makes the following main contributions:

- i. First emulation-based design for distributed event-based state estimation (EBSE) replicating a centralised discrete-time linear observer.
- ii. Stability proofs for the resulting distributed and switching estimator dynamics under generic communication or computation imperfections (bounded disturbances).
- iii. Extension to distributed event-based control, where local estimates are used for feedback.
- iv. Experimental validation on an unstable networked control system.

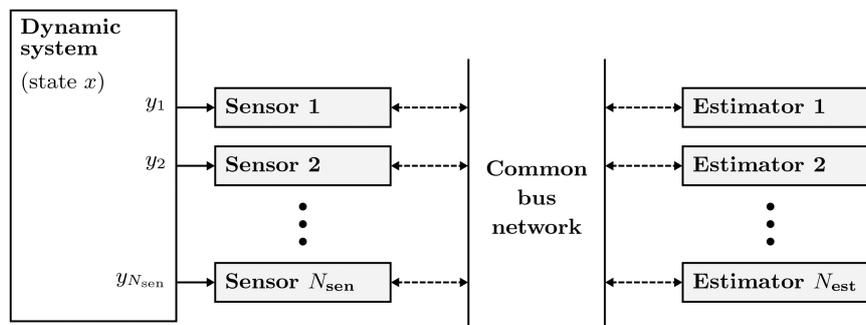


Fig. 1 Distributed state estimation problem. Multiple distributed sensors make observations y_i of a dynamic system and communicate to estimator nodes via a common bus network. Development of an event-based scheme allowing all estimators to estimate the full system state x , but with limited inter-agent communication, is the objective of this paper

Preliminary results of those herein were presented in the conference papers [3, 4]; this paper has been completely rewritten and new results added.

1.1 Related work

Early work on EBSE concerned problems with a single sensor and estimator node (see [1] and references therein). Typically, the optimal triggering strategies have time-varying thresholds for finite-horizon problems, and constant thresholds for infinite-horizon problems, [1, p. 340]. Since long-term behaviour (stability) is of primary interest herein, we consider constant thresholds.

Different types of stationary triggering policies have been suggested in the literature. With the *send-on-delta* (SoD) protocol [5], transmissions are triggered based on the difference of the current and last-transmitted measurement. *Innovation-based triggering* [6] places a threshold on the measurement innovation; i.e. the difference of the current measurement and its prediction based on a process model. Wu *et al.* [7] use the same trigger, but apply a transformation to decorrelate the innovation. Considering the variance of the innovation instead yields *variance-based triggering* [8]. Marck and Sijs [9] proposed *relevant sampling*, where the relative entropy of prior and posterior state distribution is employed as a measure of information gain. We use innovation-based triggers herein, which have been shown to be effective for EBSE [10].

Different estimation algorithms have been proposed for EBSE, with particular emphasis on how to (approximately) incorporate information contained in ‘negative’ events (instants when no data is transmitted) [11–13]. If one ignores the extra information from negative events in favour of a straightforward implementation, a time-varying Kalman filter (KF) can be used (e.g., [6]). Herein, we use the same structure as the standard KF, but with pre-computed switching gains, thus achieving the lowest computational complexity of all mentioned algorithms.

To the best of the author’s knowledge, *distributed* EBSE with multiple sensor/estimator nodes and general coupled dynamics was first studied in [6]. While Yook *et al.* [14] have previously proposed the use of state estimators for the purpose of saving communication, they do not employ state estimation in the usual sense. Instead of *fusing* model-based predictions with incoming data, they *reset* parts of the state vector. Later results on distributed EBSE include [13, 15–19]. In contrast to the scenario herein, they consider either a centralised fusion node, or simpler SoD-type triggers, which are less effective for estimation [10]. None of the mentioned references treats the problem of emulating a centralised observer design with a distributed and event-triggered implementation.

When the event-based state estimators are connected to state-feedback controllers (discussed in Section 5), this represents a *distributed event-based control* system. Wang and Lemmon [20] and Mazo and Tabuada [21] were among the first to discuss distributed or decentralised event-based control. In contrast to these works, we neither assume perfect state measurements, nor a centralised controller as in [21], nor have a restriction on the dynamic couplings [20], but we rely on a common bus network supporting all-to-all communication.

1.2 Notation

The terms *state observer* and *state estimator* are used synonymously in this paper. \mathbb{R} , \mathbb{N} , and \mathbb{N}_N denote real numbers, positive integers, and the set $\{1, 2, \dots, N\}$, respectively. Where convenient, vectors are expressed as tuples (v_1, v_2, \dots) , where v_i may be vectors themselves, with dimension and stacking clear from context. For a vector v and matrix A , $\|v\|$ denotes some vector Hölder norm [22, p. 344], and $\|A\|$ the induced matrix norm. For a sequence $v = \{v(0), v(1), \dots\}$, $\|v\|_\infty$ denotes the ℓ^∞ norm $\|v\|_\infty := \sup_{k \geq 0} \|v(k)\|$. For an estimate of $x(k)$ computed from measurement data until time $\ell \leq k$, we write $\hat{x}(k|\ell)$; and use $\dot{x}(k) = \hat{x}(k|k)$. A matrix is called stable if all its

eigenvalues have magnitude strictly less than one. Expectation is denoted by $\mathbb{E}[\cdot]$.

2 Problem statement: distributed state estimation with reduced communication

We introduce the considered networked dynamic system and state the estimation problem addressed in this paper.

2.1 Networked dynamic system

We consider the networked estimation scenario in Fig. 1. The dynamic system is described by linear discrete-time dynamics

$$x(k) = Ax(k-1) + Bu(k-1) + v(k-1) \quad (1)$$

$$y(k) = Cx(k) + w(k) \quad (2)$$

with sampling time T_s , state $x(k) \in \mathbb{R}^n$, control input $u(k) \in \mathbb{R}^q$, measurement $y(k) \in \mathbb{R}^p$, disturbances $v(k) \in \mathbb{R}^n$, $w(k) \in \mathbb{R}^p$, and all matrices of corresponding dimensions. We assume that (A, B) is stabilisable and (A, C) is detectable. No specific assumptions on the characteristics of the disturbances $v(k)$ and $w(k)$ are made; they can be random variables or deterministic disturbances.

Each of the N_{sen} sensor agents (cf. Fig. 1) observes part of the dynamic process through measurements $y_i(k) \in \mathbb{R}^{p_i}$, $i \in \mathbb{N}_{N_{\text{sen}}}$. The vector $y(k)$ thus represents the collective measurements of all N_{sen} agents

$$y(k) = (y_1(k), y_2(k), \dots, y_{N_{\text{sen}}}(k)) \quad (3)$$

$$y_i(k) = C_i x(k) + w_i(k) \quad \forall i \in \mathbb{N}_{N_{\text{sen}}} \quad (4)$$

with $C_i \in \mathbb{R}^{p_i \times n}$ and $w_i(k) \in \mathbb{R}^{p_i}$. Agents can be heterogeneous with different types and dimensions of measurements, and no local observability assumption is made (i.e., (A, C_i) can be not detectable).

Each of the N_{est} estimator agents (cf. Fig. 1) shall reconstruct the full state for the purpose of, e.g. having full information at different monitoring stations, distributed optimal decision making, or local state-feedback control. Overall, there are $N = N_{\text{sen}} + N_{\text{est}}$ agents, and we use $i = 1, \dots, N_{\text{sen}}$ to index the sensor agents, and $i = 1 + N_{\text{sen}}, \dots, N_{\text{est}} + N_{\text{sen}}$ for the estimator agents.

While the primary concern is the development of an event-based approach to the *distributed state estimation* problem in Fig. 1, we shall also address *distributed control* when the local estimates are used for feedback. For this, we consider the control input decomposed as

$$u(k) = (u_1(k), u_2(k), \dots, u_{N_{\text{est}}}(k)) \quad (5)$$

with $u_i(k) \in \mathbb{R}^{q_i}$ the input computed on estimator agent $i + N_{\text{sen}}$.

All agents are connected over a common-bus network; i.e. if one agent communicates, all agents will receive the data. We assume that the network bandwidth is such that, in the worst case, all agents can communicate in one time step T_s , and contention is resolved via low-level protocols. Moreover, agents are assumed to be synchronised in time, and network communication is abstracted as instantaneous.

Remark 1: The common bus is a key component of the developed event-based approach. It will allow the agents to compute consistent estimates and use these for effective triggering decisions (while inconsistencies can still happen due to data loss or delay). Wired networks with a shared bus architecture such as Controller Area Network (CAN) or other fieldbus systems are common in industry [23]. Recently, Ferrari *et al.* [24] have

proposed a common bus concept also for multi-hop low-power wireless networks.

2.2 Reference design

We assume that a centralised, discrete-time state estimator design is given, which we seek to emulate with the event-based design to be developed herein:

$$\hat{x}_c(k|k-1) = A\hat{x}_c(k-1|k-1) + Bu(k-1) \quad (6)$$

$$\hat{x}_c(k|k) = \hat{x}_c(k|k-1) + L(y(k) - C\hat{x}_c(k|k-1)) \quad (7)$$

where the estimator gain $L \in \mathbb{R}^{n \times p}$ has been designed to achieve desired estimation performance, and the estimator is initialised with some $\hat{x}_c(0) = \hat{x}_c(0|0)$. For example, (6) and (7) can be a KF representing the optimal Bayesian estimator for Gaussian noise, or a Luenberger observer designed via pole placement to achieve a desired dynamic response. At any rate, a reasonable observer design will ensure stable estimation error dynamics

$$\begin{aligned} e_c(k) &:= x(k) - \hat{x}_c(k) \\ &= (I-LC)Ae_c(k-1) \\ &\quad + (I-LC)v(k-1) - Lw(k). \end{aligned} \quad (8)$$

We thus assume that $(I-LC)A$ is stable, which is always possible since (A,C) is detectable. It follows [25, pp. 212–213] that there exist $m_c > 0$ and $\rho_c \in (0, 1)$ such that

$$\|((I-LC)A)^k\| \leq m_c \rho_c^k. \quad (9)$$

2.3 Problem statement

The main objective of this paper is an EBSE design that approximates the reference design of Section 2.2 with guaranteed bounds:

Problem 1: Develop a distributed EBSE design for the scenario in Fig. 1, where each estimator agent ($i = N_{\text{sen}} + 1, \dots, N_{\text{sen}} + N_{\text{est}}$) locally computes an estimate $\hat{x}_i(k)$ of the state $x(k)$, and each sensor agent ($i = 1, \dots, N_{\text{sen}}$) makes individual transmit decisions for its local measurements $y_i(k)$. The design shall emulate the centralised estimator (6) and (7) bounding the difference $\|\hat{x}_c(k) - \hat{x}_i(k)\|$, but with reduced communication of sensor measurements.

Furthermore, we address distributed control based on the EBSE design:

Problem 2: Design distributed control laws for computing control inputs $u_i(k)$ (cf. (5)) locally from the event-based estimates $\hat{x}_i(k)$ so as to achieve stable closed-loop dynamics (bounded x).

For state estimation in general, both the measurement signal y and the control input u must be known (cf. (6) and (7)). For simplicity, we first focus on the reduction of sensor measurements and assume

Assumption 1: The input u is known by all agents.

This is the case, e.g. when estimating a process without control input (i.e., $u = 0$), when u is an a-priori known reference signal, or when u is broadcast periodically over the shared bus. In particular, if the components $u_i(k)$ are computed by different agents as in Problem 2, Assumption 1 requires the agents to exchange their inputs over the bus at every step k . Reducing measurement communication, but periodically exchanging inputs may be a viable solution when there are more measurements than control inputs (as is the case for the experiment presented in Section 6.2).

Later, in Section 5, an extension of the results is presented, which does not require Assumption 1 and periodic exchange of inputs by employing event-triggering protocols also for the inputs.

3 EBSE with a single sensor–estimator link

To develop the main ideas of the EBSE approach, we first consider Problem 1 for the simpler, but relevant special case with $N_{\text{sen}} = N_{\text{est}} = 1$; i.e. a single sensor transmits data over a network link to a remote estimator (also considered in [1, 7, 9–11], for instance). For the purpose of this section, we make the simplifying assumption of a perfect communication link:

Assumption 2: Transmission from sensor to estimator is instantaneous and no data is lost.

For a sufficiently fast network link, this may be ensured by low-level protocols using acknowledgments and re-transmissions. However, this assumption is made for the sake of simplicity in this section, and omitted again in the later sections.

We propose the event-based architecture depicted in Fig. 2a. The key idea is to replicate the remote state estimator at the sensor; the sensor agents then knows what the estimator knows, and thus also when the estimator is in need of new data. The *State Estimator* and *Event Triggerer*, which together form the EBSE algorithm, are explained next.

3.1 State estimator

Both sensor and remote agents implement the state estimator (cf. Fig. 2a). The estimator recursively computes an estimate $\hat{x}_i(k) = \hat{x}_i(k|k)$ of the system state $x(k)$ from the available measurements:

$$\hat{x}_i(k|k-1) = A\hat{x}_i(k-1|k-1) + Bu(k-1) \quad (10)$$

$$\hat{x}_i(k|k) = \hat{x}_i(k|k-1) + \gamma(k)L(y(k) - C\hat{x}_i(k|k-1)) \quad (11)$$

with $i = 1$ for the sensor, $i = 2$ for the estimator, L as in (7), and $\gamma(k) \in \{0, 1\}$ denoting the sensor's decision of transmitting $y(k)$ ($\gamma(k) = 1$), or not ($\gamma(k) = 0$).

By Assumption 2, both estimators have the same input data. If, in addition, they are initialised identically, both estimates are identical, i.e. $\hat{x}_1(k) = \hat{x}_2(k)$ for all k . Hence, the sensor has knowledge about the estimator and can exploit this for the triggering decision.

3.2 Event triggerer

The sensor transmits a measurement if, and only if, the remote estimator cannot predict the measurement accurately enough based on its state prediction. Specifically, $y(k)$ is transmitted when the remote prediction $\hat{y}(k) = C\hat{x}_2(k|k-1)$ deviates from $y(k)$ by more than a tolerable threshold $\delta^{\text{est}} \geq 0$. Since $\hat{x}_1(k|k-1) = \hat{x}_2(k|k-1)$, the sensor can make this decision without requiring communication from the remote estimator:

$$\begin{aligned} \text{transmit } y(k) &\Leftrightarrow \gamma(k) = 1 \\ &\Leftrightarrow \|y(k) - C\hat{x}_1(k|k-1)\| \geq \delta^{\text{est}}. \end{aligned} \quad (12)$$

Tuning δ^{est} allows the designer to trade off the sensor's frequency of events (and, hence, the communication rate) for estimation performance. This choice of the trigger will be instrumental in bounding the difference between the event-based and the centralised estimator, as will be seen in the subsequent stability analysis. The trigger is also called *innovation-based trigger* and was previously proposed in different contexts in [6, 7, 14]. The innovation trigger (12) can also be realised without the local state estimator on the sensor by periodically communicating estimates from the remote estimator to the sensor. However, the proposed architecture avoids this additional communication.

3.3 Stability analysis

The estimator update (10) and (11) and the triggering rule (12) together constitute the proposed event-based state estimator. The estimator (10) and (11) is a switching observer, whose switching

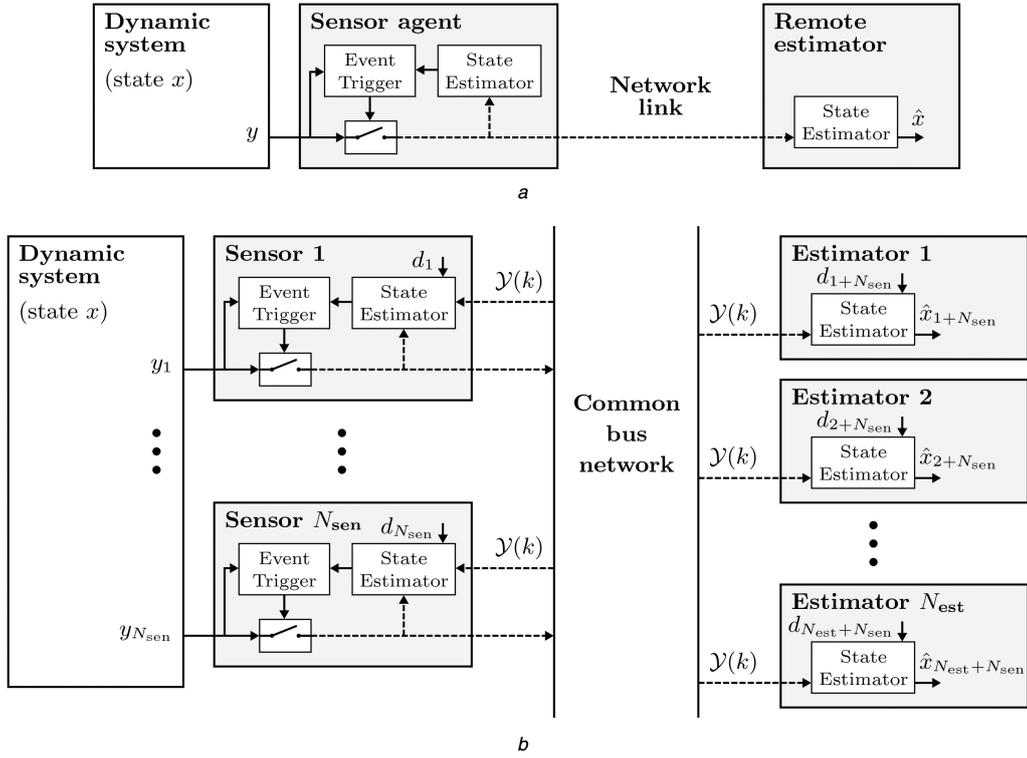


Fig. 2 Proposed EBSE architectures. Dashed arrows indicate event-based communication, while solid ones indicate periodic communication

(a) Single sensor/estimator case: sensor agent implements a copy of the remote *State Estimator* to trigger a data transmission (*Event Trigger*) whenever an update is needed at the remote agent, (b) Multiple sensor/estimator case: each agent implements a copy of the *State Estimator* for making transmit decisions (sensors) or having full state information available (estimators). The common bus supports data exchange between all agents; $\mathcal{Y}(k)$ denotes the set of measurements communicated at time k . Disturbances d_i model differences in the agents' estimates, e.g. from imperfect communication.

modes are governed by the event trigger (12). For arbitrary switching, stability of the switching observer is *not* implied by stability of the centralised design (see, e.g., [26]). Hence, proving stability is an essential, non-trivial requirement for the event-based design.

3.3.1 Difference to centralised estimator: Addressing Problem 1, we first prove a bounded difference to the centralised reference estimator $\hat{x}_c(k)$. Using (6), (7), (10), and (11), the difference $e_i(k) = \hat{x}_c(k) - \hat{x}_i(k)$ can be written as

$$\begin{aligned} e_i(k) &= A e_i(k-1) + L(y(k) - C\hat{x}_i(k|k-1)) \\ &\quad - \gamma(k)L(y(k) - C\hat{x}_i(k|k-1)) \\ &= (I - LC)A e_i(k-1) + (1 - \gamma(k)) \\ &\quad \times L(y(k) - C\hat{x}_i(k|k-1)) \end{aligned} \quad (13)$$

where the last equation was obtained by adding and subtracting $L(y(k) - C\hat{x}_i(k|k-1))$. The error $e_i(k)$ is governed by the stable centralised estimator dynamics $(I - LC)A$ with an extra input term, which is bounded by the choice of the event-trigger (12): for $\gamma(k) = 0$, $L(y(k) - C\hat{x}_i(k|k-1))$ is bounded by (12), and for $\gamma(k) = 1$, the extra term vanishes. We thus have the following result:

Theorem 1: Let Assumptions 1 and 2 be satisfied, $(I - LC)A$ be stable, and $\hat{x}_i(0) = \hat{x}_2(0) = x_0$ for some $x_0 \in \mathbb{R}^n$. Then, the difference $e_i(k)$ between the centralised estimator and the EBSE (10)–(12) is bounded by

$$\|e_i\|_\infty \leq m_c \|e_i(0)\| + \frac{m_c}{1 - \rho_c} \|L\| \delta^{\text{est}} =: e_i^{\text{max}}. \quad (14)$$

Proof: From the assumptions, it follows that $\hat{x}_1(k) = \hat{x}_2(k)$ and $\hat{x}_1(k|k-1) = \hat{x}_2(k|k-1)$. From the previous argument, we have

$$\|(1 - \gamma(k))L(y(k) - C\hat{x}_i(k|k-1))\| \leq \|L\| \delta^{\text{est}}. \quad (15)$$

The bound (14) then follows from [25, p. 218, Theorem 75] and exponential stability of $e_i(k) = (I - LC)A e_i(k-1)$ [cf. (9)]. \square

The first term in (14), $m_c \|e_i(0)\|$, is due to possibly different initial conditions between the EBSE and the centralised estimator, and $m_c \|L\| \delta^{\text{est}} / (1 - \rho_c)$ represents the asymptotic bound. Choosing δ^{est} small enough, $e_i(k)$ can hence be made arbitrarily small as $k \rightarrow \infty$, and, for $\delta^{\text{est}} = 0$, the performance of the centralised estimator is recovered.

The bound (14) holds irrespective of the nature of the disturbances v and w in (1), (2) (no assumption on v , w is made in Theorem 1). In particular, it also holds for the case of unbounded disturbances such as Gaussian noise.

3.3.2 Estimation error: The actual estimation error e_i of agent i is

$$\begin{aligned} e_i(k) &:= x(k) - \hat{x}_i(k) \\ &= e_c(k) + e_i(k). \end{aligned} \quad (16)$$

Theorem 1 can be used to deduce properties of the estimation error e_i from properties of the centralised estimator. We exemplify this for the case of bounded, as well as stochastic disturbances v and w .

Corollary 1: Let $\|v\|_\infty \leq v^{\text{max}}$, $\|w\|_\infty \leq w^{\text{max}}$, $\|e_i\|_\infty \leq e_i^{\text{max}}$ be bounded, and $(I - LC)A$ be stable. Then, the event-based estimation error (16) is bounded by

$$\|e_i\|_\infty \leq e_c^{\text{max}} + e_i^{\text{max}} \quad (17)$$

with

$$e_c^{\text{max}} := m_c \|e_c(0)\| + \frac{m_c}{1 - \rho_c} (\|I - LC\| v^{\text{max}} + \|L\| w^{\text{max}}).$$

Proof: The bound e_c^{\max} on the centralised estimation error $e_c(k)$ follows directly from (8), exponential stability (9), and [25, p. 218, Theorem 75]. The result (17) is then immediate from (16). \square

Corollary 2: Let $v, w, x(0)$ be random variables with $\mathbb{E}[v(k)] = 0$, $\mathbb{E}[w(k)] = 0$, $\mathbb{E}[x(0)] = x_0$, and the centralised estimator be initialised with $\hat{x}_c(0) = x_0$. Let $\|e_i\|_\infty \leq e_i^{\max}$ be bounded, and $(I - LC)A$ be stable. Then, the expected event-based estimation error (16) is bounded by

$$\| \mathbb{E}[e_i(k)] \| \leq e_i^{\max} \quad \forall k. \quad (18)$$

Proof: From (8), it follows $\mathbb{E}[e_c(k)] = (I - LC)A\mathbb{E}[e_c(k-1)]$, and thus $\mathbb{E}[e_c(k)] = 0$ by recursion from $\mathbb{E}[e_c(0)] = \mathbb{E}[x(0)] - x_0 = 0$. Therefore

$$\| \mathbb{E}[e_i(k)] \| = \| \mathbb{E}[e_i(k)] \| \leq \mathbb{E}[\|e_i(k)\|] \leq e_i^{\max} \quad (19)$$

where the first inequality follows from Jensen's inequality, and the last from $\|e_i(k)\| \leq e_i^{\max}$. \square

4 EBSE with multiple agents

We extend the ideas of the previous section to the general multi-agent case in Problem 1. While the assumption of perfect communication (Assumption 2) may possibly be realisable for few agents, it becomes unrealistic as the number of agents increases. Thus, we generalise the stability analysis to the case where agents' estimates can differ.

4.1 Architecture

We propose the distributed event-based architecture depicted in Fig. 2b for the multi-agent problem. Adopting the key idea of the single agent case (cf. Fig. 2a), each agent implements a copy of the state estimator for making transmit decisions. The common bus network ensures that, if a measurement is transmitted, it is broadcast to all other units. For the estimators to be consistent, the sensor agents also listen to the measurement data $\mathcal{Y}(k)$ broadcast by other units.

The proposed EBSE scheme is *distributed* in the sense that data from distributed sensors is required for stable state estimation, and that transmit decisions are made locally by each agent.

4.2 Event trigger

In analogy to the single agent case (12), each agent i , $i \in \mathbb{N}_{N_{\text{sen}}}$, uses the following event triggering rule:

$$\text{transmit } y_i(k) \Leftrightarrow \|y_i(k) - C_i \hat{x}_i(k|k-1)\| \geq \delta_i^{\text{est}}. \quad (20)$$

The prediction $\hat{y}_i(k) = C_i \hat{x}_i(k|k-1)$ computed by agent i is representative also for all other agents' predictions of the same measurement, $\hat{y}_j^i(k) = C_j \hat{x}_i(k|k-1)$, as long as $\hat{x}_i(k|k-1) \simeq \hat{x}_j(k|k-1)$, which is to be established in the stability analysis below. Being able to approximately represent the other agents' knowledge is the basis for making effective transmit decisions in the proposed approach.

For later reference, we introduce $\delta^{\text{est}} := (\delta_1^{\text{est}}, \dots, \delta_{N_{\text{sen}}}^{\text{est}})$ and the index sets of transmitting and not-transmitting agents:

$$I(k) := \{i \in \mathbb{N}_{N_{\text{sen}}} \mid \|y_i(k) - C_i \hat{x}_i(k|k-1)\| \geq \delta_i^{\text{est}}\} \quad (21)$$

$$\begin{aligned} I^c(k) &:= \{i \in \mathbb{N}_{N_{\text{sen}}} \mid \|y_i(k) - C_i \hat{x}_i(k|k-1)\| < \delta_i^{\text{est}}\} \\ &= \mathbb{N}_{N_{\text{sen}}} \setminus I(k). \end{aligned} \quad (22)$$

4.3 State estimator

Extending the event-based estimator (10) and (11) to the multi-sensor case, we propose the following estimator update for all agents ($i \in \mathbb{N}_N$):

$$\hat{x}_i(k|k-1) = A\hat{x}_i(k-1|k-1) + Bu(k-1) \quad (23)$$

$$\hat{x}_i(k|k) = \hat{x}_i(k|k-1) + \sum_{\ell \in I(k)} L_\ell (y_\ell(k) - C_\ell \hat{x}_i(k|k-1)) \quad (24)$$

where $L_\ell \in \mathbb{R}^{n \times p_\ell}$ is the submatrix of the centralised gain $L = [L_1, L_2, \dots, L_{N_{\text{sen}}}]$ corresponding to y_ℓ . Rewriting (7) as

$$\begin{aligned} \hat{x}_c(k|k) &= \hat{x}_c(k|k-1) \\ &+ \sum_{\ell \in \mathbb{N}_{N_{\text{sen}}}} L_\ell (y_\ell(k) - C_\ell \hat{x}_c(k|k-1)) \end{aligned} \quad (25)$$

we see that (24) is the same as the centralised update, but only updating with a subset $I(k) \subset \mathbb{N}_{N_{\text{sen}}}$ of all measurements. If, at time k , no measurement is transmitted (i.e., $I(k) = \emptyset$), then the summation in (24) vanishes; i.e. $\hat{x}_i(k|k) = \hat{x}_i(k|k-1)$.

To account for differences in any two agents' estimates, e.g. from unequal initialisation, different computation accuracy, or imperfect communication, we introduce a generic disturbance signal d_i acting on each estimator (cf. Fig. 2b). For the stability analysis, we thus replace (24) with

$$\begin{aligned} \hat{x}_i(k|k) &= \hat{x}_i(k|k-1) \\ &+ \sum_{\ell \in I(k)} L_\ell (y_\ell(k) - C_\ell \hat{x}_i(k|k-1)) + d_i(k). \end{aligned} \quad (26)$$

The disturbances are assumed to be bounded:

Assumption 3: For all $i \in \mathbb{N}_N$, $\|d_i\|_\infty \leq d_i^{\max}$.

This assumption is realistic, when d_i represent imperfect initialisation or different computation accuracy, for example. Even though the assumption may not hold for modelling packet drops in general, the developed method was found to be effective also for this case in the example of Section 6.1.

4.4 Stability analysis

We discuss stability of the distributed EBSE system given by the process (1) and (4), the (disturbed) estimators (23) and (26), and the triggering rule (20). We first consider the difference between the centralised and event-based estimate, $e_i(k) = \hat{x}_c(k) - \hat{x}_i(k)$. By straightforward manipulation using (6), (23), (25), and (26), we obtain

$$\begin{aligned} e_i(k) &= Ae_i(k-1) \\ &+ \sum_{\ell \in \mathbb{N}_{N_{\text{sen}}}} L_\ell (y_\ell(k) - C_\ell \hat{x}_c(k|k-1)) \\ &- \sum_{\ell \in I(k)} L_\ell (y_\ell(k) - C_\ell \hat{x}_i(k|k-1)) - d_i(k) \\ &= (I - LC)Ae_i(k-1) \\ &+ \sum_{\ell \in I^c(k)} L_\ell (y_\ell(k) - C_\ell \hat{x}_i(k|k-1)) - d_i(k) \\ &= (I - LC)Ae_i(k-1) \\ &+ \sum_{\ell \in I^c(k)} L_\ell (y_\ell(k) - C_\ell \hat{x}_\ell(k|k-1)) \\ &- d_i(k) - \sum_{j \in I^c(k)} L_j C_j A e_j(k-1) \end{aligned} \quad (27)$$

where $e_{ij}(k) := \hat{x}_i(k) - \hat{x}_j(k)$ is the inter-agent error, and we used $\hat{x}_i(k|k-1) - \hat{x}_\ell(k|k-1) = Ae_{i\ell}(k-1)$. The error dynamics (28) are governed by stable dynamics $e_i(k) = (I - LC)Ae_i(k-1)$ with three input terms. The term $\sum_{\ell \in \mathcal{I}^c(k)} L_\ell(y_\ell(k) - C_\ell \hat{x}_\ell(k|k-1))$ is analogous to the last term in (13) and bounded by the event triggering (20) (cf. (22)). The last two terms are due to the disturbance d_i and resulting inter-agent differences e_{ij} . To bound e_i , e_{ij} must also be bounded, which is established next.

4.4.1 Inter-agent error: The inter-agent error can be written as

$$\begin{aligned} e_{ij}(k) &= \hat{x}_i(k) - \hat{x}_j(k) = Ae_{ij}(k-1) \\ &+ \sum_{\ell \in I(k)} L_\ell(y_\ell(k) - C_\ell \hat{x}_\ell(k|k-1)) + d_i(k) \\ &- \sum_{\ell \in I(k)} L_\ell(y_\ell(k) - C_\ell \hat{x}_j(k|k-1)) - d_j(k) \\ &= \tilde{A}_{I(k)} e_{ij}(k-1) + d_i(k) - d_j(k) \end{aligned} \quad (29)$$

where \tilde{A}_J is defined for some subset $J \subseteq \mathbb{N}_{N_{\text{sen}}}$ by

$$\tilde{A}_J := (I - \sum_{\ell \in J} L_\ell C_\ell)A. \quad (30)$$

Hence, the inter-agent error $e_{ij}(k)$ is governed by the time-varying dynamics $e_{ij}(k) = \tilde{A}_{I(k)} e_{ij}(k-1)$. Unfortunately, one cannot, in general, infer stability of the inter-agent error (and thus the event-based estimation error (28)) from stability of the centralised design. A counterexample is presented in [4].

A sufficiency result for stability of the inter-agent error can be obtained by considering the dynamics (29) under arbitrary switching; i.e. with \tilde{A}_J for all subsets $J \subseteq \mathbb{N}_{N_{\text{sen}}}$. The following result is adapted from [27, Lemma 3.1].

Lemma 1: Let Assumption 3 hold, and let the matrix inequality

$$\tilde{A}_J^T P \tilde{A}_J - P < 0 \quad (31)$$

be satisfied for some positive definite $P \in \mathbb{R}^{n \times n}$ and for all subsets $J \subseteq \mathbb{N}_{N_{\text{sen}}}$. Then, for given initial errors $e_{ij}(0)$ ($i, j \in \mathbb{N}_N$), there exists $e^{\max} \in \mathbb{R}$, $e^{\max} \geq 0$, such that

$$\|e_{ij}\|_\infty \leq e^{\max}, \quad \text{for all } i, j \in \mathbb{N}_N \quad (32)$$

and the Euclidean norm $\|\cdot\|$.

Proof: Under (31), the error dynamics (29) are input-to-state stable (ISS) following the proof of [27, Lemma 3.1] ($A_{ci}(J)$ replaced with \tilde{A}_J). With Assumption 3, ISS guarantees boundedness of the inter-agent error e_{ij} and thus the existence of $e_{ij}^{\max} \geq 0$ (possibly dependent on the initial error $e_{ij}(0)$) such that

$$\|e_{ij}\|_\infty \leq e_{ij}^{\max}. \quad (33)$$

Finally, (32) is obtained by taking the maximum over all e_{ij}^{\max} . \square

The stability test is conservative because the event trigger (20) will generally not permit arbitrary switching. Since $J \subseteq \mathbb{N}_{N_{\text{sen}}}$ also includes the empty set (i.e., $\tilde{A}_\emptyset = A$), the test can only be used for open-loop stable dynamics (1). In Section 4.4.3, we present an alternative approach to obtained bounded e_{ij} for arbitrary systems.

4.4.2 Difference to centralised estimator: With the preceding lemma, we can now establish boundedness of the estimation error (28).

Theorem 2: Let Assumptions 1 and 3 and the conditions of Lemma 1 be satisfied, and let $(I - LC)A$ be stable. Then, the difference $e_i(k)$ between the centralised estimator and the EBSE (20), (23), (26) is bounded by

$$\begin{aligned} \|e_i\|_\infty &\leq m_c \|e_i(0)\| + \frac{m_c}{1 - \rho_c} \\ &\times (\|L\| \|\delta^{\text{est}}\| + d_i^{\max} + \bar{m} N_{\text{sen}} e^{\max}) =: e_i^{\max} \end{aligned} \quad (34)$$

with m_c, ρ_c as in (9), and $\bar{m} := \max_{j \in \mathbb{N}_{N_{\text{sen}}}} \|L_j C_j A\|$.

Proof: We can establish the following bounds (for all k):

$$\left\| \sum_{\ell \in \mathcal{I}^c(k)} L_\ell(y_\ell(k) - C_\ell \hat{x}_\ell(k|k-1)) \right\| \stackrel{(22)}{\leq} \|L\| \|\delta^{\text{est}}\| \quad (35)$$

$$\|d_i(k)\| \stackrel{\text{Ass. 3}}{\leq} d_i^{\max} \quad (36)$$

$$\begin{aligned} &\left\| \sum_{j \in \mathcal{I}^c(k)} L_j C_j A e_{ij}(k-1) \right\| \\ &\leq \sum_{j \in \mathcal{I}^c(k)} \|L_j C_j A\| \|e_{ij}(k-1)\| \stackrel{\text{Lemma 1}}{\leq} \bar{m} N_{\text{sen}} e^{\max}. \end{aligned} \quad (37)$$

The result (34) then follows from (28), stability of $(I - LC)A$, and [25, p. 218, Thm. 75]. \square

4.4.3 Synchronous estimator resets: We present a straightforward extension of the event-based communication scheme, which guarantees stability even if the inter-agent error dynamics (29) cannot be shown to be stable (e.g., if Lemma 1 does not apply).

Since the inter-agent error $e_{ij}(k)$ is the difference between the state estimates by agents i and j , we can make it zero by resetting the two agents' state estimates to the same value, e.g. their average. Therefore, a straightforward way to guarantee bounded inter-agent errors is to periodically reset all agents' estimates to their joint average. Clearly, this strategy increases the communication load on the network. If, however, the disturbances d_i are small or only occur rarely, the required resetting period can typically be large relative to the underlying sampling time T_s .

We assume that the resetting happens after all agents have made their estimator updates (26). Let $\hat{x}_i(k-)$ and $\hat{x}_i(k+)$ denote agent i 's estimate at time k before and after resetting, and let $K \in \mathbb{N}$ be the fixed resetting period. Each agent i implements the following synchronous averaging:

$$\begin{aligned} \text{If } k \text{ a multiple of } K: & \quad \text{transmit } \hat{x}_i(k-); \\ & \quad \text{receive } \hat{x}_j(k-), j \in \mathbb{N}_N \setminus \{i\}; \\ & \quad \text{set } \hat{x}_i(k+) = \sum_{j=1}^N \hat{x}_j(k-). \end{aligned} \quad (38)$$

We assume that the network capacity is such that the mutual exchange of the estimates can happen in one time step, and no data is lost in the transfer. In other scenarios, one could take several time steps to exchange all estimates, at the expense of a delayed reset. The synchronous averaging period K can be chosen from simulations assuming a model for the inter-agent disturbances d_i (e.g., packet drops).

We have the following stability result for EBSE with synchronous averaging (38).

Theorem 3: Let Assumptions 1 and 3 be satisfied and $(I - LC)A$ be stable. Then, the difference $e_i(k)$ between the centralised estimator and the EBSE with synchronous averaging given by (20), (23), (26), and (38) is bounded.

Proof: Since the agent error (28) is affected by the resetting (38), we first rewrite $e_i(k)$ in terms of the average estimate $\bar{x}(k) := \text{avg}(\hat{x}_i(k)) := \frac{1}{N} \sum_{i=1}^N \hat{x}_i(k)$. Defining $\bar{e}(k) := \hat{x}_c(k) - \bar{x}(k)$ and $\bar{e}_i(k) := \bar{x}(k) - \hat{x}_i(k)$, we have $e_i(k) = \bar{e}(k) + \bar{e}_i(k)$ and will establish the claim by showing boundedness of $\bar{e}_i(k)$ and $\bar{e}(k)$ separately.

For the average estimate $\bar{x}(k)$, we obtain from (23), (26),

$$\begin{aligned} \bar{x}(k|k-1) &= A\bar{x}(k-1) + Bu(k-1) \\ \bar{x}(k) &= \bar{x}(k|k-1) \\ &\quad + \sum_{\ell \in I(k)} L_\ell(y_\ell(k) - C_\ell \bar{x}(k|k-1)) + \bar{d}(k) \end{aligned}$$

where $\bar{x}(k|k-1) := \text{avg}(\hat{x}_i(k|k-1))$ and $\bar{d}(k) := \text{avg}(d_i(k))$. The dynamics of the error $\bar{e}_i(k)$ are described by

$$\bar{e}_i(k) = \tilde{A}_{I(k)} \bar{e}_i(k-1) + \bar{d}(k) - d_i(k) \quad (39)$$

$$\bar{e}_i(k+) = 0, \quad \text{for } k = \kappa K \text{ with some } \kappa \in \mathbb{N} \quad (40)$$

where (39) is obtained by direct calculation analogous to (29), and (40) follows from (38). Since $\bar{d}(k)$, $d_i(k)$, and $\tilde{A}_{I(k)}$ are all bounded, boundedness of \bar{e}_i for all $i \in \mathbb{N}_N$ follows.

Since $\bar{e}(k) = \text{avg}_i(e_i(k))$, we obtain from (28)

$$\begin{aligned} \bar{e}(k) &= (I - LC)A\bar{e}(k-1) \\ &\quad + \sum_{\ell \in I^c(k)} L_\ell(y_\ell(k) - C_\ell \hat{x}_\ell(k|k-1)) - \bar{d}(k) \\ &\quad - \sum_{j \in I^c(k)} L_j C_j A \bar{e}_j(k-1) \end{aligned} \quad (41)$$

where we used $\text{avg}_i(e_{ij}(k)) = \text{avg}_i(\hat{x}_i(k) - \hat{x}_j(k)) = \bar{x}(k) - \hat{x}_j(k) = \bar{e}_j(k)$. Note that (41) fully describes the evolution of $\bar{e}(k)$. In particular, the resetting (38) does not affect $\bar{e}(k)$ because, at $k = \kappa K$, it holds

$$\begin{aligned} \bar{e}(k+) &= \hat{x}_c(k) - \frac{1}{N} \sum_{j=1}^N \hat{x}_j(k+) \\ &= \hat{x}_c(k) - \frac{1}{N} \sum_{j=1}^N \left(\frac{1}{N} \sum_{\ell=1}^N \hat{x}_\ell(k-) \right) \\ &= \hat{x}_c(k) - \frac{1}{N} \sum_{\ell=1}^N \hat{x}_\ell(k-) = \bar{e}(k-). \end{aligned} \quad (42)$$

All input terms in (41) are bounded: \bar{d} by Assumption 3, $\sum_{\ell \in I^c(k)} L_\ell(y_\ell(k) - C_\ell \hat{x}_\ell(k|k-1))$ by (22), and \bar{e}_j by the previous argument. The claim then follows from stability of $(I - LC)A$. \square

4.4.4 Estimation error: By means of (16) with Theorem 2 or Theorem 3, properties about the agent's estimation error $e_i(k) = x(k) - \hat{x}_i(k)$ can be derived given properties of the disturbances v , w , and the centralised estimator. For example, Corollaries 1 and 2 apply analogously also for the multi-agent case.

5 Distributed control

In this section, we address Problem 2; i.e. the scenario where the local estimates \hat{x}_i on the N_{est} estimators are used for feedback control.

Recall the decomposition (5) of the control input, where $u_i(k)$ is the input computed on estimator agent $i + N_{\text{sen}}$. Assume a centralised state-feedback design is given

$$u(k) = Fx(k) \quad (43)$$

with controller gain $F \in \mathbb{R}^{q \times n}$ such that $A + BF$ is stable. We propose the distributed state-feedback control law

$$u_i(k) = F_i \hat{x}_{i+N_{\text{sen}}}(k), \quad i \in \mathbb{N}_{N_{\text{est}}} \quad (44)$$

where $F_i \in \mathbb{R}^{q_i \times n}$ is the part of the gain matrix F in (43) corresponding to the local input u_i . Same as for the emulation-based estimator design in the previous sections, the feedback gains do not need to be specifically designed, but can simply be taken from the centralised design (43).

5.1 Closed-loop stability analysis

Using (16) and (44), the state equation (1) can be rewritten as

$$\begin{aligned} x(k) &= (A + BF)x(k-1) \\ &\quad - \sum_{i \in \mathbb{N}_{N_{\text{est}}}} B_i F_i e_{i+N_{\text{sen}}}(k-1) + v(k-1) \end{aligned} \quad (45)$$

where $e_{i+N_{\text{sen}}}(k-1)$ are the estimation errors of the estimator agents (cf. Section 4.4.4). Closed-loop stability can then be deduced leveraging the results of Section 4.

Theorem 4: Let the assumptions of either Theorem 2 or Theorem 3 be satisfied, $A + BF$ be stable, and v and w bounded. Then, the state of the closed-loop control system given by (1), (2), (20), (23), (26), (44), and (possibly) (38) is bounded.

Proof: Since $(I - LC)A$ is stable and v , w bounded, it follows from (8) that the estimation error $e_c(k)$ of the centralised observer is also bounded. Thus, (16) and Theorem 2 or Theorem 3 imply that all e_i , $i \in \mathbb{N}_N$, are bounded. Hence, it follows from (45), stability of $A + BF$, and bounded v that x is also bounded. \square

Satisfying Assumption 1 for the above result requires the periodic communication of all inputs over the bus. While this increases the network load, it can be a viable option if the number of inputs is comparably small. Next, we briefly present an alternative scheme, where the communication of inputs is reduced also by means of event-based protocols.

5.2 Event-based communication of inputs

Each estimator agent computes $u_i(k)$ according to (44) and broadcasts an update to the other agents whenever there has been a significant change:

$$\text{transmit } u_i(k) \Leftrightarrow \|u_i(k) - u_{i,\text{last}}(k)\| \geq \delta_i^{\text{ctrl}} \quad (46)$$

where $\delta_i^{\text{ctrl}} \geq 0$ is a tuning parameter, and $u_{i,\text{last}}(k)$ is the last input that was broadcast by agent i .

Each agent i maintains an estimate $\hat{u}^i(k) \in \mathbb{R}^q$ of the complete input vector $u(k)$; agent i 's estimate of agent j 's input is

$$\hat{u}_j^i(k) = \begin{cases} u_j(k) & \text{if (46) triggered} \\ \hat{u}_j^i(k-1) & \text{otherwise.} \end{cases} \quad (47)$$

The agent then uses $\hat{u}^i(k-1) = (\hat{u}_1^i(k-1), \hat{u}_2^i(k-1), \dots, \hat{u}_N^i(k-1))$ instead of the true input $u(k-1)$ for the estimator update (23). Since the error $\hat{u}^i(k) := u(k) - \hat{u}^i(k)$ from making this approximate update is bounded by the event trigger (46), the stability results presented in Section 4 can be extended to this case. The details are omitted, but can be found in [28].

6 Experiments

To illustrate the proposed approach for EBSE, we present numerical simulations of a benchmark problem [29] and

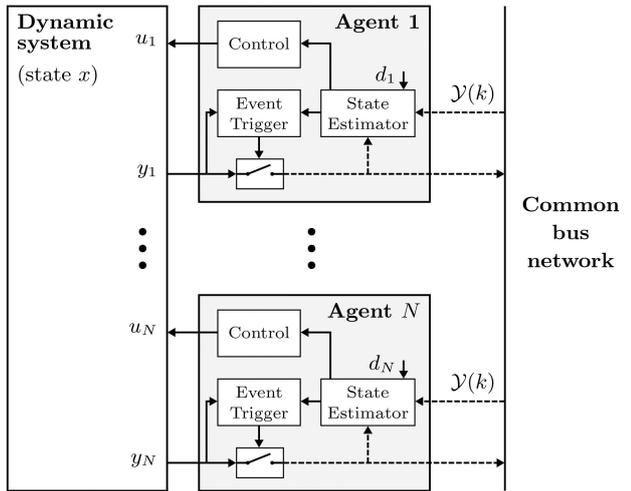


Fig. 3 Distributed, event-based control architecture in the experiments of Section 6. N spatially distributed agents observe and control a dynamic system and exchange data with each other via a common bus. Compared to Fig. 2b, the agents combine the functions of sensing/triggering, estimation, and additionally feedback control. The State Estimator serves both for making the triggering decision and for feedback control

summarise experimental results from [3]. Both are examples of the multi-agent case where local estimates are used for control.

6.1 Simulation of a thermo-fluid benchmark process

We consider distributed event-based control of a thermo-fluid process, which has been proposed as a benchmark problem in [29, 30]. Matlab files to run the simulation example are provided as supplementary material (<http://is.tue.mpg.de/publications/tr17>).

The process has two tanks containing fluids, whose level and temperature are to be regulated by controlling the tanks' inflows, as well as heating and cooling units. Both tanks are subject to step-like disturbances, and their dynamics are coupled through cross-flows between the tanks. Each tank is associated with a control agent responsible for computing commands to the respective actuators. Each agent can sense the temperature and level of its tank. For details on the process, refer to [29, 30].

6.1.1 System description: The discrete-time linear model (1) and (2) is obtained by zero-order hold discretisation with $T_s = 0.2$ s of the continuous-time model given in [29, Section 5.8]. The process

Table 1 States and inputs of the thermo-fluid process

States/Inputs		Unit
$x_1(k)$	level tank 1	m
$x_2(k)$	temperature tank 1	K
$x_3(k)$	level tank 2	m
$x_4(k)$	temperature tank 2	K
$u_{11}(k)$	inflow tank 1	1 (normalised)
$u_{12}(k)$	cooling tank 1	1 (normalised)
$u_{21}(k)$	inflow tank 2	1 (normalised)
$u_{22}(k)$	heating tank 2	1 (normalised)

Table 2 Agents in the thermo-fluid example. Agent 1 measures $y_1 = (y_{11}, y_{12})$ (level and temperature) of its tank and is responsible for controlling $u_1 = (u_{11}, u_{12})$ (inflow and cooling); and Agent 2 accordingly

Agent #	1	2
actuators	$u_1 = (u_{11}, u_{12})$	$u_2 = (u_{21}, u_{22})$
sensors	$y_1 = (y_{11}, y_{12})$	$y_2 = (y_{21}, y_{22})$

dynamics are stable. The states and inputs of the system are summarised in Table 1. Noisy state measurements

$$y(k) = x(k) + w(k)$$

are available, where $w(k)$ is uniformly distributed. The numerical parameters of the model are available in the supplementary files.

Similar to the distributed architecture in [30], we consider two agents (one for each tank) exchanging data with each other over a network link, see Fig. 3 (with $N = 2$) and Table 2 for inputs/output definitions. Each agent combines the functions of sensing, estimation, and control. To save computational resources, an agent runs a single estimator and uses it for both event triggering (20) and feedback control (44) (see [6] for an alternative architecture with two estimators).

To study the effect of imperfect communication, we simulate random packet drops such that a transmitted measurement $y_i(k)$ is lost with probability 0.05, independent of previous drops. Packet drops can be represented by the disturbance $d_i(k)$ in (26) as follows: if $y_{\ell}(k)$, $\ell \in I(k)$ is a measurement not received at agent i , then $d_i(k) = -L_{\ell}(y_{\ell}(k) - C_{\ell}\hat{x}_i(k|k-1))$ accounts for the lost packet. For simplicity, we assume that communicated inputs are never lost.

6.1.2 Event-based design: Each agent implements the event triggers (20) and (46), the estimator (23) and (24), and the distributed control (44). Triggering decisions are made individually for the two sensors of each agent, but jointly for both inputs (cf. Table 2).

For the design of the centralised observer (6), (7), we chose $L = \text{diag}(0.1, 0.05, 0.1, 0.05)$ as observer gain, leading to stable $(I - LC)A$. For this design, the inter-agent error dynamics (29) are also stable: by direct calculation, one can verify that (31) is satisfied with $P = \text{diag}(500, 1, 500, 1)$ for all subsets $J \subseteq \{1, 2, 3, 4\}$ (cf. supplementary material). Lemma 1 thus guarantees that (29) is stable, and synchronous resetting (38) not necessary.

The state-feedback gain F is obtained from a linear quadratic regulator (LQR) design, which involves full couplings between all states in contrast to the decentralised design in [30]. The triggering thresholds are set to $\delta_{11}^{\text{est}} = \delta_{21}^{\text{est}} = 0.01$ m, $\delta_{12}^{\text{est}} = \delta_{22}^{\text{est}} = 0.2$ K, and $\delta_1^{\text{ctrl}} = \delta_2^{\text{ctrl}} = 0.02$.

6.1.3 Simulation results: The state trajectories of a 2000 s simulation run under event-based communication are shown in Fig. 4. Step-wise disturbances v (grey shaded areas) with comparable magnitudes as in [30] cause the states to deviate from zero. Especially at times when disturbances are active, the event-based estimate is slightly inferior to the centralised one, as is expected due to the reduced number of measurements.

The average communication rates for event-based input and sensor transmissions are given in Fig. 5. Clearly, communication rates increase in the periods where the disturbances are active, albeit not the same for all sensors and inputs. At times when there is no disturbance, communication rates are very low.

Fig. 6 shows the inter-agent error e_{12} . Jumps in the error signals are caused by dropped packets, with decay afterward due to stable dynamics (29).

6.2 Experiments on the Balancing Cube

The proposed emulation-based approach to event-based estimation was applied in [3] for stabilising the Balancing Cube [31] (see Fig. 7). In this section, we summarise the main results from the experimental study reported in [3]. For details, we refer to these citations.

6.2.1 System description: The cube is stabilised through six rotating arms on its rigid structure (see Fig. 7). Each arm constitutes a control agent equipped with sensors (angle encoder and rate gyroscopes), a DC motor, and a computer. The computers

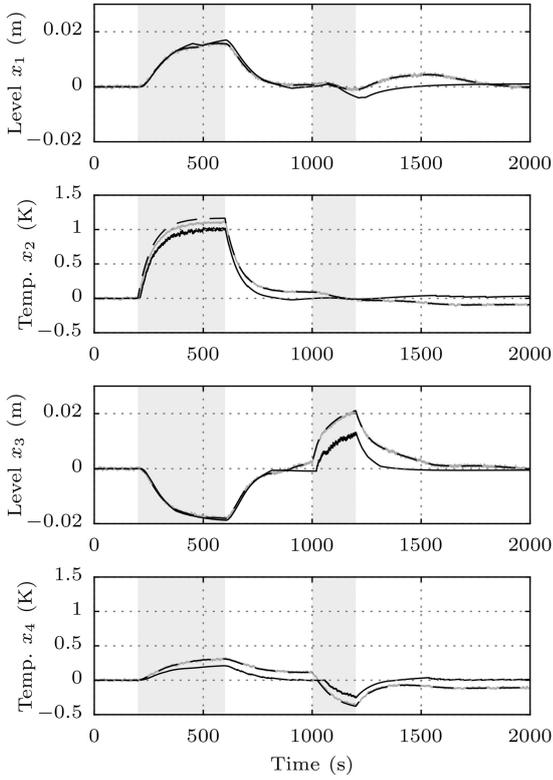


Fig. 4 State trajectories for the thermo-fluid simulation example. Black (dashed): the actual states x ; black (solid): event-based estimate \hat{x}_i by Agent i ; grey: centralised estimate \hat{x}_c . Centralised estimate is shown for comparison and not available on any of the agents. Grey shaded areas indicate periods where step-wise process disturbances are active

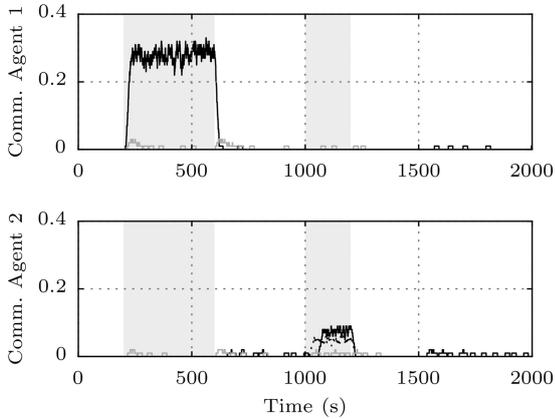


Fig. 5 Event-based communication rates for the thermo-fluid simulation example: communication of level measurements y_{11} and y_{21} in black (dotted), for temperature measurements y_{12} and y_{22} in black (solid), and for the inputs u_1 and u_2 in grey. Communication rates are computed as the moving average over 100 steps (0.0 meaning no communication and 1.0 full communication)

are connected over a CAN bus, which supports the exchange of sensor data between all agents (including the worst case of all agents communicating within one sampling time $T_s = 1/60$ s). Each agent thus combines the functions sensing, triggering, estimation, and control as shown in Fig. 3 ($N = 6$).

6.2.2 Event-based design: A model (1) and (2) representing linearised dynamics about the equilibrium configuration shown in Fig. 7 is used for designing the centralised observer (6) and (7) (as a steady-state KF) and the controller (43) (LQR). Each agent makes individual triggering decisions for its angle sensor and for

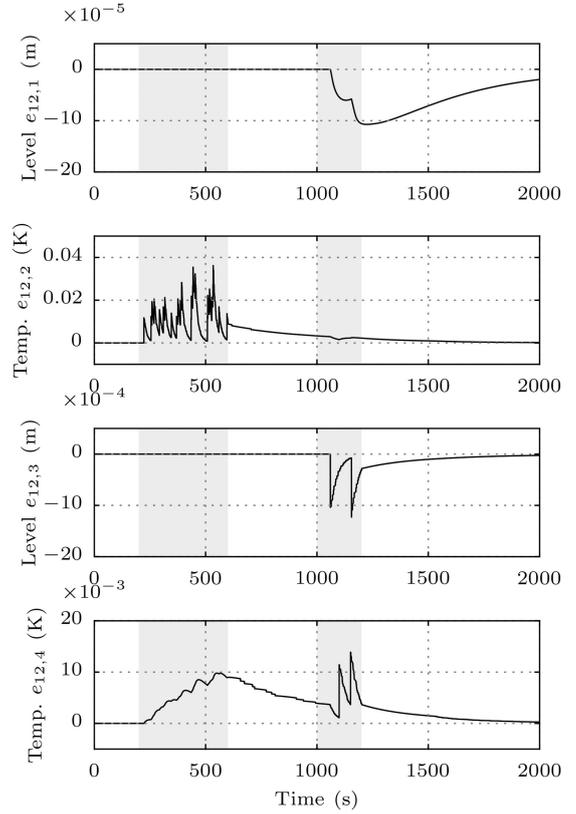


Fig. 6 Inter-agent error e_{i2} for the thermo-fluid example. Jumps in error are caused by packet drops, and the decay afterward is due to stable inter-agent dynamics (29) as ensured by Lemma 1

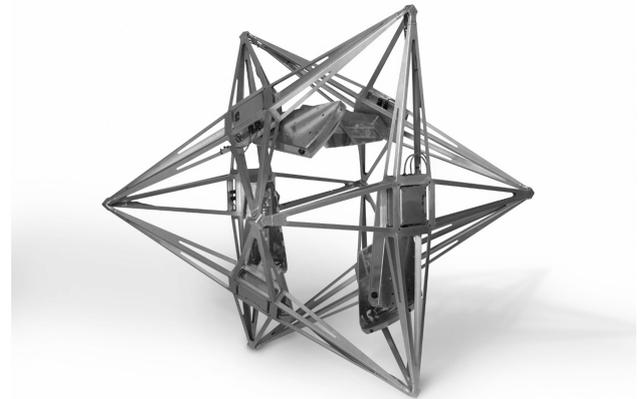


Fig. 7 The Balancing Cube [31] can balance autonomously on any one of its corners or edges. Six rotating arms, which constitute the control agents, collaboratively keep the system in balance. Here, the cube is shown while balancing on an edge, which was the configuration used for the experiments in [3]. Photo: Carolina Flores, IDSC, ETH Zurich. © 2012 IEEE. Reprinted, with permission, from [31]

its rate gyroscope with thresholds $\delta^{\text{ang}} = 0.008$ rad and $\delta^{\text{gyro}} = 0.004$ rad/s, respectively.

In the experiments, control inputs u_i were communicated periodically between all agents. Synchronous resetting (38) was *not* applied, even though stability of the inter-agent error (29) cannot be shown using Lemma 1 because of unstable open-loop dynamics. Despite the absence of a formal proof, the system was found to be stable in balancing experiments.

6.2.3 Experimental results: Fig. 8 shows typical communication rates for some sensors during balancing. The desired behaviour of event-based communication is well visible: feedback happens *only when necessary* (e.g., instability or disturbances). Overall, the

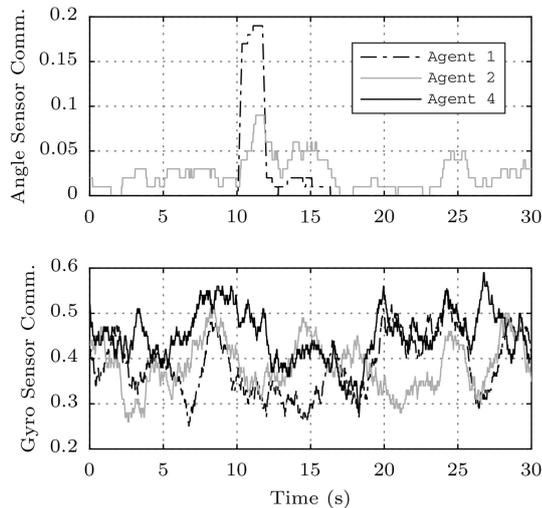


Fig. 8 Experimental communication rates on the Balancing Cube (reproduced from [3]). Communication rates (between 0 and 1) are computed as moving average over the last 100 time steps. Rate gyroscopes generally transmit at higher rates than the angle sensors since they observe the unstable mode of the system. Angle measurements can be predicted well from the process model; thus only little communication is necessary (e.g., Agent 4 does not transmit over 30 s). Caused by an external disturbance applied at 10 s on Agent 1 (pushing the arm), the communication rate of Agent 1 goes up temporarily

network traffic could be reduced by about 78% at only a mild decrease in estimation performance.

7 Concluding remarks

Simplicity of design and implementation are key features of the emulation-based approach to EBSE developed herein. The approach directly builds on a classic centralised, linear, discrete-time state observer design. Essentially, only the event triggers (20) and (46), and (for some problems) synchronous resetting (38) must be added. The estimator structure, as well as the transmitted quantities remain unchanged, and no redesign of gains is necessary. The performance of the periodic design can be recovered by choosing small enough triggering thresholds, which simplifies tuning in practise. Thus, implementation of the event-based system requires minimal extra effort, and virtually no additional design knowledge.

With the proposed event-based method, the average communication load in a networked control system can be significantly reduced, as demonstrated in the simulations and experiment in this paper.

8 Acknowledgments

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